

On the Power Spectrum Density of Gamma Ray Bursts

Motoko Suzuki and Masahiro Morikawa

and

Izumi Joichi

Department of Physics, Ochanomizu University, Tokyo 112-8610, Japan

School of Science and Engineering, Teikyo University, Toyosatodai 1-1, Utsunomiya 320-8551, Japan

ABSTRACT

Gamma ray bursts (GRBs) are known to have short-time variability and power-law behavior with the index -1.67 in the power spectrum density. Reanalyzing the expanded data, we have found a) the power-law comes from the global profile of the burst and not from the self-similar shots nor rapid fluctuations in the luminosity profile. b) The power indices vary from burst to burst and the value -1.67 is given simply as the mean value of the distribution; there is no systematic correlation among GRBs to yield the power law.

Subject headings: gamma ray, burst, power spectrum density, power-law

1. INTRODUCTION

The origin and the fundamental mechanism of the gamma-ray bursts (GRBs) have not been revealed despite the fact that more than thirty years have passed since the first discovery of them. In the numerous luminosity profiles of GRBs, we notice that most GRBs have very rapid milli-second time variability in their intensity (Schaefer & Walker 1999) though there is vast variety in luminosity profile itself for each GRB. We believe that this characteristic variability must have important information to reveal the GRB mechanism. Therefore, in this letter, we would like to concentrate on the analysis of this variability.

In studying time sequence of objects, the Fourier transform technique is useful and often yields indispensable information on the scaling properties and characteristic time scale. For example, Beloborodov et al. (1998) used this method for 214 light curves of long GRBs ($T_{90} > 20$ sec) and reported that the averaged power spectrum density (PSD) of GRBs shows the power-law with the index 1.67 ± 0.02 over two decades in frequency range. This value is very closed to $-5/3$ which suggests the Kolmogorov spectrum of velocity fluctuations in turbulent medium.

On the other hand, the power-law in PSD does not necessarily specify the whole mechanism as we know in various examples. Actually the power-law in PSD can be derived by many reasons; some specific burst profile in luminosity, Levy-type random noise in the background, and the superposition of similar shots and so on. For example, the power-law in PSD of X-ray emission from the Active Galactic Nuclei (AGN) is considered to be originated from the superposition of many similar shots

(Mineshige & Yonehara 2001). Chang & Yi (2000) tried to reproduce the power-law index, $-5/3$, and the individual PSD distribution of GRBs by the superposition of many decaying pulse shots.

Here in this paper, we would like to determine the origin of the power-law in PSD of GRBs by analyzing the detail of PSD for each burst data.

First, we demonstrate that the individual PSD of GRBs does not exactly have the power-law index -1.67 but has wide variation which is well approximated by the Gaussian distribution. Second, we argue that power-law index of PSD is determined by the profiles of an individual shot in the light curve and not by the superposition of many similar shots nor by Levy-type random noise. Then we discuss the general nature of PSD of superposed shots and apply this method to the actual data of GRB light curves. Finally we observe how the averaged PSD shows clear power-law index -1.67 .

2. POWER LAW IN PSD OF GRB

In our analysis, we use the data detected by the Burst and Transient Source Experiment (BATSE) on Compton Gamma Ray Observatory (CGRO) with 64ms resolution ¹. We use the light curves in the energy band $50 < h\nu < 100$ keV, excluding unclear high and low energy bands, and subtract the background by the linear fitting. Then for excluding the noise contamination, we select the data with peak count rate larger than 250 counts per 64ms bin; finally 297 data set remain.

In order to see the generality of the power index $-5/3$, we first calculate the power spectra, which is the absolute square of the Fourier transform of the time sequence, for individual light curves. We assume the following fitting function for the power spectrum $P(f)$, expecting the coexistence of the power-law component and the thermal white-noise component.

$$P(f) = Af^\alpha + B. \quad (1)$$

Figure 1 shows the distribution of power-law index α thus obtained. The distribution is well fitted by the Gaussian and the mean value is -1.76 , which is close to the value -1.67 reported in the reference Beloborodov et al. (1998). We emphasize here that the individual data has the power-law PSD though the power index is widely distributed with the variance 0.65.

3. WHAT DETERMINES THE INDIVIDUAL POWER LAW?

Now let us consider the meaning of the power-law in PSD. As is well known, the power-law itself in the PSD can be realized in various origin. The specifications of the origin is significant for the analysis of the central engine of GRBs.

Mineshige & Yonehara (2001) shows at least the following three ways to realize the power-law in PSD.

1. Specific global profile of a burst.

¹ftp://coss.gsfc.nasa.gov/pub/data/batse/ascii_data/64ms/

2. Levy-type random process.
3. Superposition of self-similar shots.

We examine the above possibilities in the following in this order.

Examining the first possibility, we realize that each GRB data generally has multiple shots and rapid fluctuations as well as noise whose origin is not seem to be GRB. We try to identify these components in each GRB data step by step.

First we discuss the light curve with a single shot. This simple type of light curve shows a clear power-law behavior in its PSD. We consider the following two model cases; a) the decay type, and b) the grow-and-decay type.

a) The decay type

The decay type shot has the flux x at time t as

$$x(t) = ht^{-p} \exp(-f_0 t) \theta(t), \quad (2)$$

where the positive parameters h , p and f_0 respectively represent “intensity”, “sharpness” and “inverse of the duration” of the shot. $\theta(t)$ is the step function. Power spectrum P for this shot is written as

$$P(f) = h^2 [\Gamma(1-p)]^2 [f_0^2 + f^2]^{p-1}, \quad (3)$$

where $\Gamma(x)$ is the Gamma function.

b) The grow-and-decay type

The grow-and-decay type shot generally has asymmetric profile in the burst; we separate the grow (left side) and decay (right side) of the shot. Each side of the shot is written as

$$x(t) = h_L (-t)^{-p_L} \exp(t f_{0L}) \theta(-t) + h_R (t)^{-p_R} \exp(-t f_{0R}) \theta(t) \quad (4)$$

Power spectrum of this type is written as

$$\begin{aligned} P(f) = & h_L^2 \{\Gamma(1-p_L)\}^2 (f_{0L}^2 + f^2)^{p_L-1} + h_R^2 \{\Gamma(1-p_R)\}^2 (f_{0R}^2 + f^2)^{p_R-1} \\ & + 2h_L h_R \Gamma(1-p_L) \Gamma(1-p_R) \\ & \times (f_{0L}^2 + f^2)^{\frac{p_L-1}{2}} (f_{0R}^2 + f^2)^{\frac{p_R-1}{2}} \cos[(p_L-1)\theta_L + (p_R-1)\theta_R], \end{aligned} \quad (5)$$

where $\theta_{L,R} = \arctan(f/f_{0L,R})$.

Since the GRB data generally has multiple shots, the above single shot profiles must be superposed before we use. For simplicity for fitting procedure, we assume that all shots in the individual light curve has the same profile except the overall amplitude²; i.e. the k -th shot in the light curve $x_k(t)$ is written as $A_k x(t - a_k)$, where A_k is the relative amplitude and a_k is the location of the k -th shot with

²When we fit actual data of GRBs, we use inverse of duration f_0 as a fitting parameter, but it does not change the index of PSD at all.

$x(t)$ being the fixed function for each GRB data. Then we can easily calculate the PSD for multiple shots since the Fourier transforms of the first and the k -th shots are simply related with each other as $\tilde{x}_k = A_k e^{i f a_k} \tilde{x}_1$.

The light curve $x_T(t)$ which has n shots

$$x_T(t) = \sum_{k=1}^n x_k(t), \quad (6)$$

has the PSD

$$P_T(f) = P(f) \left[\sum_{j=1}^n A_j^2 + \sum_{j>k} 2A_j A_k \cos f(a_j - a_k) \right], \quad (7)$$

where $P(f)$ is PSD for $x(t)$.

It is apparent that the last expression consists of the “constant” term and the “oscillating” term; the former term determines the global profile of PSD and the latter fluctuations around it.

We can actually see this structure in PSD of GRB data. We first identify the maximum shot in the GRB data and locally fit this shot by the single-shot form argued in the above. Then we subtract this first fit from the original data yielding one-shot subtracted data. Second, we identify the maximum shot in this subtracted data and locally fit this shot by the single-shot form with the same parameter p . Then we subtract this second fit from the one-shot subtracted data yielding two-shots subtracted data. After repeating this process several times, the reduced data becomes almost flat.

We applied this method for many individual GRB data. One typical example for GRB910602(#257) is shown in Fig.2. Here we use the decay type shot for fitting identifying two shots in the data. The full PSD of this light curve is plotted in Fig.3 with the solid line. The PSD of the first fit is plotted with the chain line which already shows smooth power-law. The PSD of the superposition of the first and the second fits is plotted with the broken line which shows the oscillation around the chain line. It is almost clear that the PSD of the superposition of first few fits are sufficient to faithfully reproduce the main part of the original PSD. Other GRB data sets show similar behavior as this one.

Thus we have confirmed the first possibility: Specific global profile of a burst determines the power-law behavior in PSD.

In order to exclude the second possibility, the Levy-type random process, we have examined the peak distribution analysis. We calculate the distribution function of all peak-intensity in the data of artificially produced Levy-type light curve and that in the actual data of GRBs. The former shows a clear power law however the latter does not. We can now claim that the power-law in PSD of GRBs is not originated from the Levy-type random process.

In order to exclude the third possibility i.e. superposition of self-similar shots, we calculate the PSD of the few-shots subtracted data. For the previous data set, we calculate the PSD of the two-shots subtracted data in Fig.3 with the dotted line. This residual component is 10-100 times smaller than the main component and is almost flat. If GRBs have self-similar structure in their light curves, the few-shots subtracted data should also show power law (but its range must be narrower than that of

original data). Thus we can now claim that the power-law in PSD of GRBs is not originated from the superposition of self-similar shots

We have also applied the same analysis to the data set of the soft gamma-ray repeaters (SGR). We use the data of SGR1900+14 ³. The first one-shot fit has been enough to reproduce the power-law behavior in its original PSD ⁴ and the one-shot subtracted data shows a flat PSD except several very sharp peaks which are considered to be originated from the central pulsar of SGR.

4. WHAT DETERMINES THE AVERAGED POWER LAW?

We have averaged all the 297 PSD of GRB data and obtain the Figure 4. Before taking the average, we have normalized each GRB luminosity data so that the maximum of the count rate be unity. This is because the original light curves have two to three orders of difference in maximum count rates and therefore the naively averaged PSD is determined by few GRB data. Since the magnitude of the count rate itself is not physical (we don't know the distance!), such discrimination is not a proper manipulation. We have also tried the total-count-rate (fluence) normalization as well as maximum-count-rate normalization; yielding no significant difference between them.

Thus averaged PSD shows much clear power-law (Fig.4) than the individual PSD of GRB data. From Fig.4 we again observe that the individual PSD shows power-law and the power index simply fluctuates variously. After the superposition of many PSD data, there appears smooth power-law behavior with the power index of the central value. It is important to observe in Fig. 4 that there is no systematic correlation in each PSD to yield the global smooth power-law behavior. Especially there is no systematic distribution of time scales nor correlation of turning points which, if any, would have yielded clear envelope when many PSD are superposed.

The above reasoning for the appearance of the smooth power-law in averaged PSD is also supported by the fact that the power index -1.67 which is close enough to the mean value -1.76 of the index distribution within 14 percent of the variance. If the clear power-law were realized by the envelope of many systematic distribution of burst time scales, the averaged power index would be significantly smaller than the mean power index.

5. CONCLUSIONS AND DISCUSSIONS

Analyzing 297 power spectrum density (PSD) of GRBs, we obtain the following results in this paper. a) Individual GRB data shows power law behavior in the PSD. The power index (α) distribution is well approximated by the Gaussian form with the mean -1.76 and the variance 0.65. b) Power law behavior in PSD for individual GRB data is determined by the shot profile and not by the Levy-type noise nor the superposition of many self-similar shots. c) Power law index in averaged PSD is simply determined by the mean value of the index (α) distribution. We found no correlation mechanism for

³<http://ssl.berkeley.edu/ipn3/sgr1900+14.lightcurve>

⁴Fitting parameter p is 0.045 and power index of original PSD is -1.8 .

producing clear power law in averaged PSD.

From the result b), we doubt the turbulence (Beloborodov et al. 1998, 2000) as the origin of the power law in PSD of GRBs. This is because the self-similar cascade of eddy, which would naturally yield many self-similar shots in the emission, yields the Kolmogorov power law spectrum of velocity fluctuations in turbulent medium.

The result b) provides sharp contrast with the case of power law in the X-ray emission from AGN. Many authors have demonstrated that some self-similar cascade model, which yields many self-similar shots, successfully describes the emission profile and the power law in PSD of AGN. Thus we naturally expect that the emission mechanism of AGN and that of GRB are quite different with each other despite the similar power law behavior in their PSD.

As we have seen in our analysis, some special form of shots, which yield the power law in PSD, is very characteristic and would be a good criterion for restricting various models of GRB generation mechanisms. The special shot profile is more severe checking point of the validity of the model than simply the power-law behavior of PSD (Panaitescu et al. 1999; Spada et al. 2000). An urgent interest then would be the question whether the popular "Internal shock model" can explain these special shot profiles.

There are still important questions to be answered in the near future. 1) The shots in the light curves of GRBs have distinctly different two types; the decay type and the grow-and-decay type. This property is different from that of blazars, even if their similarity in emission process is often suggested. 2) What is the distribution of the sharpness parameter p and the ratio of power-law component and noise A/B in equation (1) ? Is it also Gaussian or any other form from which we can extract the information of the distance or classification of GRB?

We all would like to thank Takashi Okamura for many useful suggestion. I. J. would also like to thank Prof. T. Yokobori and Prof. A. T. Yokobori, Jr for their hospitality.

REFERENCES

- Beloborodov, A. M., Stern, B. E., & Svensson, R. 1998, *ApJ*, 508, L25
- . 2000, *ApJ*, 535, 158
- Chang, H.-Y. & Yi, I. 2000, *ApJ*, 542, L17
- Mineshige, S. & Yonehara, A. 2001, in *Probing the Physics of Active Galactic Nuclei by Multiwavelength Monitoring*, ed. B. M. Peterson, R. S. Polidan, & R. W. Pogge
- Panaitescu, A., Spada, M., & Meszaros, P. 1999, *ApJ*, 522, L105
- Schaefer, B. E. & Walker, K. C. 1999, *ApJ*, 511, L89
- Spada, M., Panaitescu, A., & Meszaros, P. 2000, *ApJ*, 537, 824

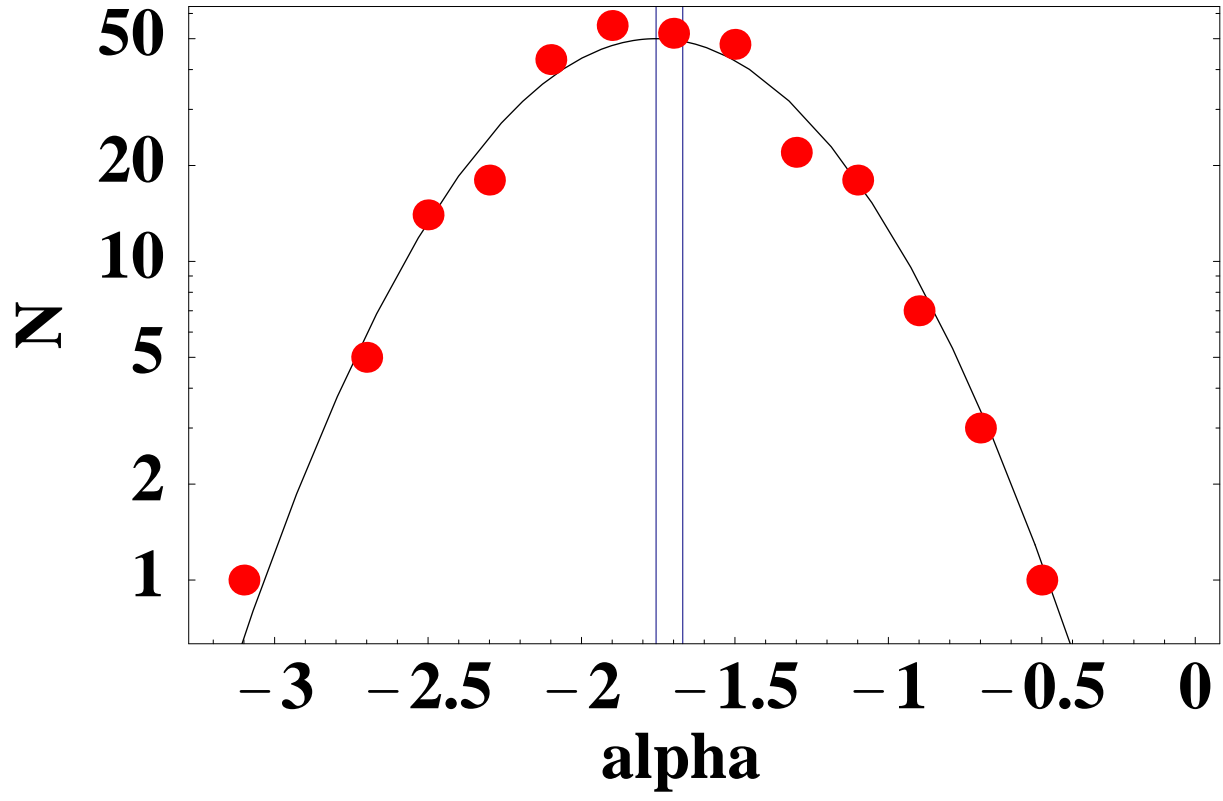


Fig. 1.— The distribution of the power index α for 297 data sets. The distribution is well fitted by the Gaussian with the mean -1.76 and the variance 0.65 .

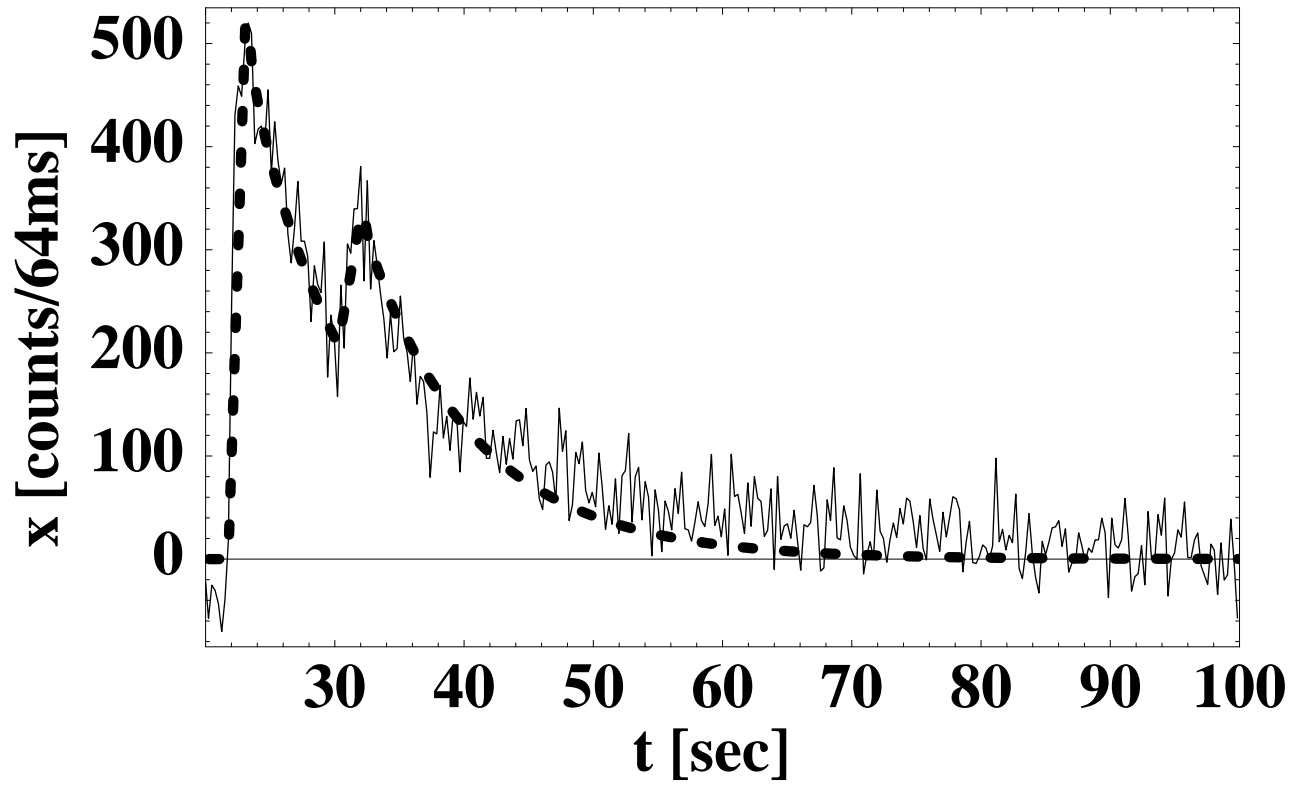


Fig. 2.— Actual light curve GRB910602(#257) (solid line) and fitting shots (broken line); the superposition of the first and the second fits. The parameter p is 0.03.

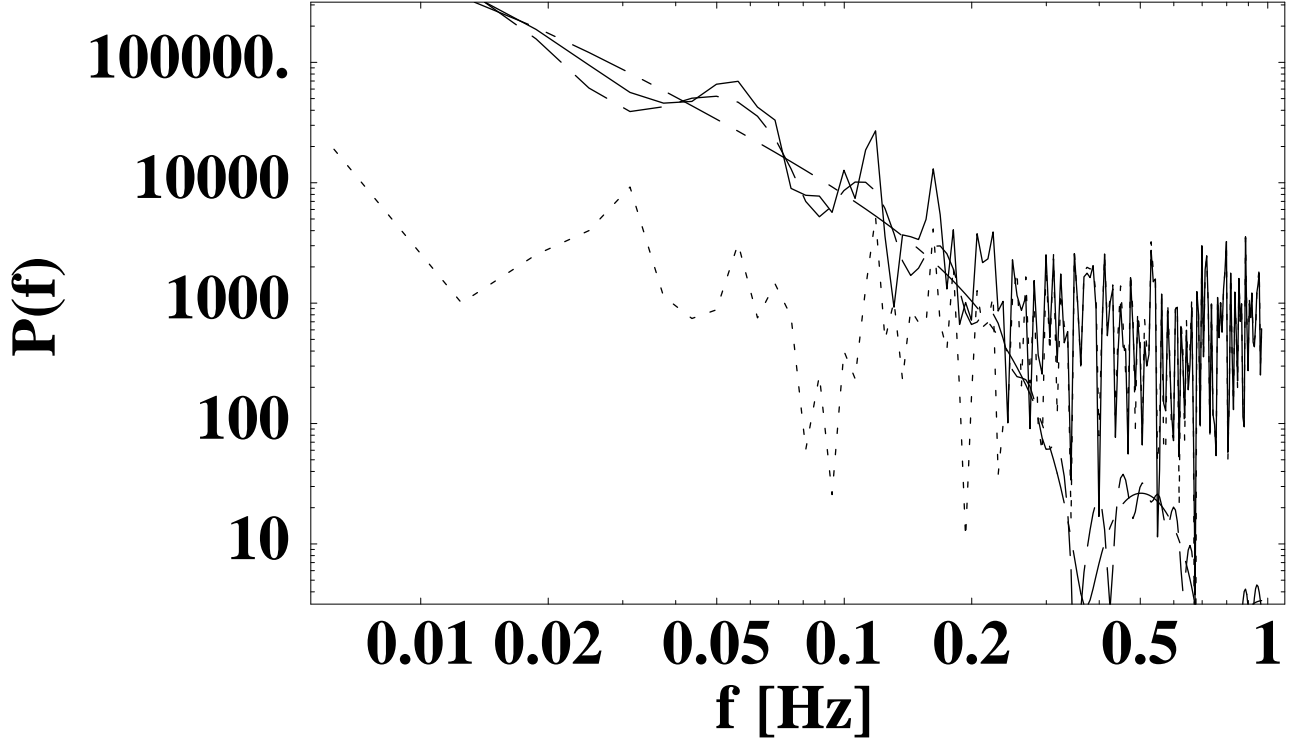


Fig. 3.— Power spectrum of the light curve in Fig.2. We plot PSD of actual data (solid line), PSD of the first fit (chain line), PSD of the superposition of the first and the second fits (broken line) and PSD of the two-shots subtracted data (dotted line). PSD of the original data is hierarchically decomposed into the first shot, second shot and the small fraction of the subtracted component; our method is successfully working. Note that the PSD of the first fit already matches well with that of the original data especially at low frequency regions. Power index of this data is -1.95 .

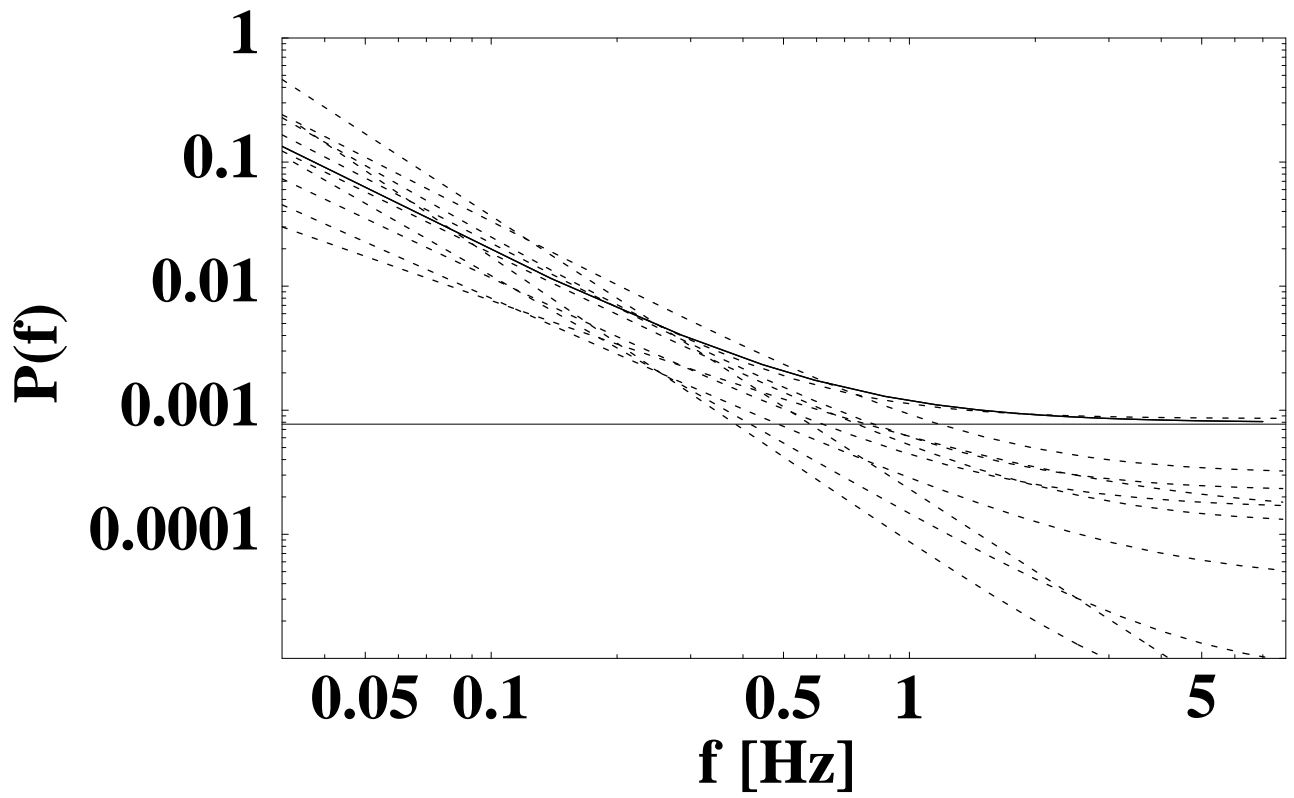


Fig. 4.— Fits of individual (dotted lines) and average (solid line) power spectra. We plot oldest 10 data of individual power spectra out of 297, and plot average of all 297 data.